1.) $y=(x^4-4x^3)^{\frac{1}{4}}$

1° $y=\sqrt[4]{x^4-4x^3}$

$D(y): x^4-4x^3 \geq 0$

$x^3(x-4) \geq 0$

	$-\infty$	0		4	$+\infty$
x	$-$	0	$+$		$+$
$x-4$	$-$		$-$	0	$+$
$x^3(x-4)$	$+$	0	$-$	0	$+$

$D(y): (-\infty, 0] \cup [4, +\infty)$

2° ni parna ni neparna

3° $y=0 \Leftrightarrow x^4-4x^3=0$

$\Leftrightarrow x^3(x-4)=0$

$\Leftrightarrow x=0 \vee x=4$ M(0,0), N(4,0)

- $y \geq 0, \forall x \in D(y)$

4° $\lim\limits_{x\to+\infty} f(x) = \lim\limits_{x\to+\infty} \sqrt[4]{x^4-4x^3} = \lim\limits_{x\to+\infty} \sqrt[4]{x^4\left(1-\frac{4}{x}\right)} = \lim\limits_{x\to+\infty} |x| \cdot \sqrt[4]{1-\cancel{\frac{4}{x}}^{\to 0}} =$

$= \lim\limits_{x\to+\infty} x \sqrt[4]{1-\frac{1}{x}} = +\infty$

$\lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to-\infty} |x| \cdot \sqrt[4]{1-\frac{4}{x}} = \lim\limits_{x\to-\infty} -x \cdot \sqrt[4]{1-\cancel{\frac{4}{x}}^{\to 0}} = +\infty$

Nema horizontalne asimptote.

$k_1 = \lim\limits_{x\to+\infty} \frac{f(x)}{x} = \lim\limits_{x\to+\infty} \frac{\cancel{x} \cdot \sqrt[4]{1-\cancel{\frac{4}{x}}^{\to 0}}}{\cancel{x}} = \underline{1}$

- $n_1 = \lim\limits_{x\to+\infty} (f(x)-k_1x) = \lim\limits_{x\to+\infty} \left(x \cdot \sqrt[4]{1-\frac{4}{x}} - x\right) \overset{\infty-\infty}{=} \lim\limits_{x\to+\infty} x\left(\sqrt[4]{1-\frac{4}{x}}-1\right)$

$\overset{\infty\cdot 0}{=} \lim\limits_{x\to+\infty} \frac{\sqrt[4]{1-\frac{4}{x}}-1}{\frac{1}{x}} \underset{L.P.}{\overset{\frac{0}{0}}{=}} \lim\limits_{x\to+\infty} \frac{\frac{1}{4}\left(1-\frac{4}{x}\right)^{-\frac{3}{4}} \cdot \left(\frac{4}{x^2}\right)}{-\frac{1}{x^2}} = -\lim\limits_{x\to+\infty} \frac{1}{\sqrt[4]{\left(1-\frac{4}{x}\right)^3}} = -1$

$y = k_1 x + n_1$

$y = x - 1 \quad K.A. \quad x \to +\infty$

$$k_2 = \lim_{x\to-\infty} \frac{f(x)}{x} = \lim_{x\to-\infty} \frac{-x\cdot\sqrt[4]{1-\frac{4}{x}}}{x} = -1$$

$$n_2 = \lim_{x\to-\infty} (f(x) - k_2 x) = \lim_{x\to-\infty} \left(-x\cdot\sqrt[4]{1-\frac{4}{x}} + x\right) \overset{\infty-\infty}{=}$$

$$= \lim_{x\to-\infty} x\left(1 - \sqrt[4]{1-\frac{4}{x}}\right) \overset{-\infty\cdot 0}{=} \lim_{x\to-\infty} \frac{1-\sqrt[4]{1-\frac{4}{x}}}{\frac{1}{x}} \overset{\frac{0}{0}}{\underset{LP}{=}}$$

$$= \lim_{x\to-\infty} \frac{-\frac{1}{4}\left(1-\frac{4}{x}\right)^{-\frac{3}{4}}\cdot\frac{4}{x^2}}{-\frac{1}{x^2}} = \lim_{x\to-\infty} \frac{1}{\sqrt[4]{\left(1-\frac{4}{x}\right)^3}} = 1$$

$y = k_2 x + n_2$

$y = -x + 1, \quad K.A. \quad x \to -\infty$

$5^\circ \quad y = (x^4 - 4x^3)^{\frac{1}{4}}$

$$y' = \frac{1}{4}\cdot(x^4-4x^3)^{-\frac{3}{4}}\cdot(4x^3-12x^2)$$

$$y' = \frac{1}{4}\cdot\frac{4x^3-12x^2}{\sqrt[4]{(x^4-4x^3)^3}} = \frac{1}{4}\cdot\frac{4x^2(x-3)}{\sqrt[4]{x^9(x-4)^3}} =$$

$$= \frac{x^2(x-3)}{x^2\cdot\sqrt[4]{x(x-4)^3}} = \frac{x-3}{\sqrt[4]{x(x-4)^3}}$$

$y' = 0 \iff x - 3 = 0$

$\iff x = 3 \notin D(y)$

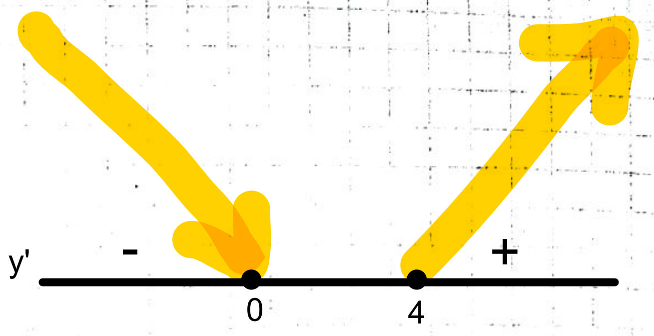
y' nije definisano u $x = 0$ i $x = 4$

$x = 0 \in D(y)$, $x = 4 \in D(y)$

$y' > 0 \iff x-3>0 \wedge x \in D(y)$

$\iff x>3 \wedge x \in D(y)$

$\iff x \in (4, +\infty)$



U x=0 minimum
U x=4 minimum

$y' < 0 \iff x-3<0 \wedge x \in D(y)$

$\iff x<3 \wedge x \in D(y)$

$\iff x \in (-\infty, 0)$

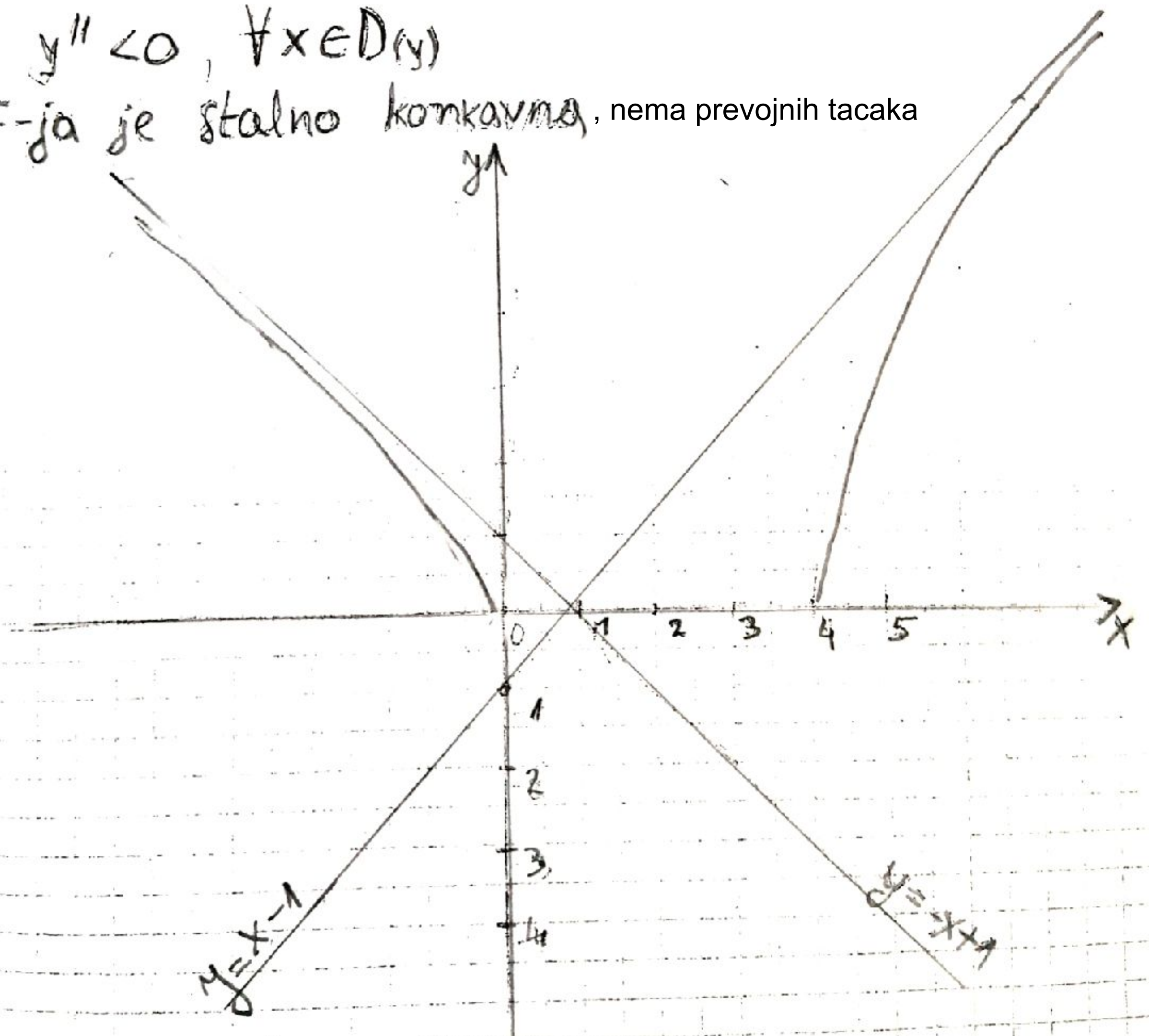
6° $y' = \dfrac{x-3}{\sqrt[4]{x(x-4)^3}}$

$y'' = \dfrac{-3}{\sqrt[4]{x^5 \cdot (x-4)^7}}$

$y'' \neq 0, \ \forall x \in D(y)$ i y'' nije def. za $x=0$ i $x=4$

$y'' < 0, \ \forall x \in D(y)$

F-ja je stalno konkavna, nema prevojnih tacaka



2.) $y = \frac{\ln|x|+1}{x}$

1° $|x|>0 \wedge x \neq 0$

$D(y): (-\infty, 0) \cup (0, +\infty)$

2° $y(-x) = \frac{\ln|-x|+1}{-x} = \frac{\ln|x|+1}{-x} = -\frac{\ln|x|+1}{x} = -y(x)$

y je neparna f-ja (grafik je simetričan u odnosu na koordinatni početak)

Dalje ćemo f-ju ispitivati na intervalu $(0, +\infty)$

$$y = \begin{cases} \frac{\ln x+1}{x}, & x>0 \\ \frac{\ln(-x)+1}{x}, & x<0 \end{cases}$$

$$y = \frac{\ln x+1}{x}, \quad x \in (0, +\infty)$$

3° $y=0 \Leftrightarrow \ln x+1=0,\ x>0$

$\Leftrightarrow \ln x=-1,\ x>0$

$\Leftrightarrow x=e^{-1}$

$\Leftrightarrow x=\frac{1}{e}$ $\quad N(\frac{1}{e}, 0)$

$y>0 \Leftrightarrow \ln x+1>0 \wedge x>0$

$\Leftrightarrow \ln x>-1 \wedge x>0$

$\Leftrightarrow \ln x>\ln e^{-1} \wedge x>0$

$\Leftrightarrow x>\frac{1}{e}$

$y<0 \Leftrightarrow \ln x+1<0 \wedge x>0$

$\Leftrightarrow \ln x<-1 \wedge x>0$

$\Leftrightarrow x<\frac{1}{e} \wedge x>0$

$\Leftrightarrow x \in (0, \frac{1}{e})$

4° $\lim\limits_{x\to 0+} f(x) = \lim\limits_{x\to 0+} \frac{\ln x+1}{x} = \lim\limits_{x\to 0+} \underbrace{\frac{1}{x}}_{+\infty} \cdot \underbrace{(\ln x+1)}_{-\infty} = -\infty$

$x=0$ V.A.

$$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} \frac{\ln x+1}{x} \overset{\frac{\infty}{\infty}}{\underset{L.P.}{=}} \lim_{x\to+\infty} \frac{\frac{1}{x}}{1} = \lim_{x\to+\infty} \frac{1}{x} = 0$$

$y=0$ H.A.

Nema kose asimptote

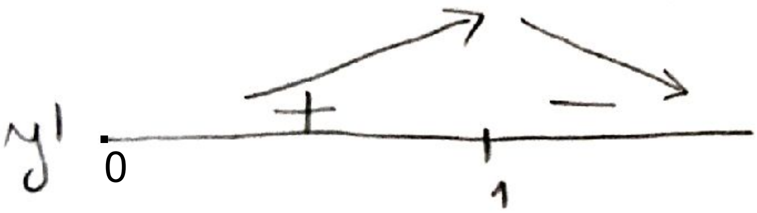
5° $$y' = \frac{\frac{1}{x}\cdot x - (\ln x+1)\cdot 1}{x^2} = \frac{1-\ln x-1}{x^2} = -\frac{\ln x}{x^2}$$

$$y'=0 \Leftrightarrow \ln x = 0,\ x>0$$
$$\Leftrightarrow x=1$$

y' nije def. za $x=0$, ali $x=0 \notin D(y)$

$$y'>0 \Leftrightarrow \ln x<0,\ x>0$$
$$\Leftrightarrow x<1,\ x>0$$
$$(\Leftrightarrow x\in(0,1))$$

$$y'<0 \Leftrightarrow \ln x>0,\ x>0$$
$$\Leftrightarrow x>1$$



U $x=1$ f postiže lok. maksimum.

$$f_{max} = f(1) = \frac{\ln 1+1}{1} = 1 \qquad M(1,1)$$

6° $$y' = -\frac{\ln x}{x^2}$$

$$y'' = -\frac{\frac{1}{x}\cdot x^2 - \ln x\cdot 2x}{x^4} = -\frac{x-2x\ln x}{x^4} = \frac{2\ln x-1}{x^3}$$

$y'' = 0 \iff 2\ln x - 1 = 0 \quad x>0$

$\iff \ln x = \frac{1}{2} \quad x>0$

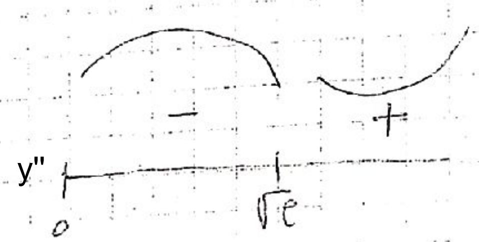
$\iff x = e^{\frac{1}{2}} = \sqrt{e}$

y'' nije definisano u $x=0$, $0 \notin D(y)$

$y'' > 0 \iff 2\ln x - 1 > 0 \wedge x>0$

$\iff \ln x > \frac{1}{2} \wedge x>0$

$\iff x > \sqrt{e}$



$y'' < 0 \iff 2\ln x - 1 < 0 \wedge x>0$

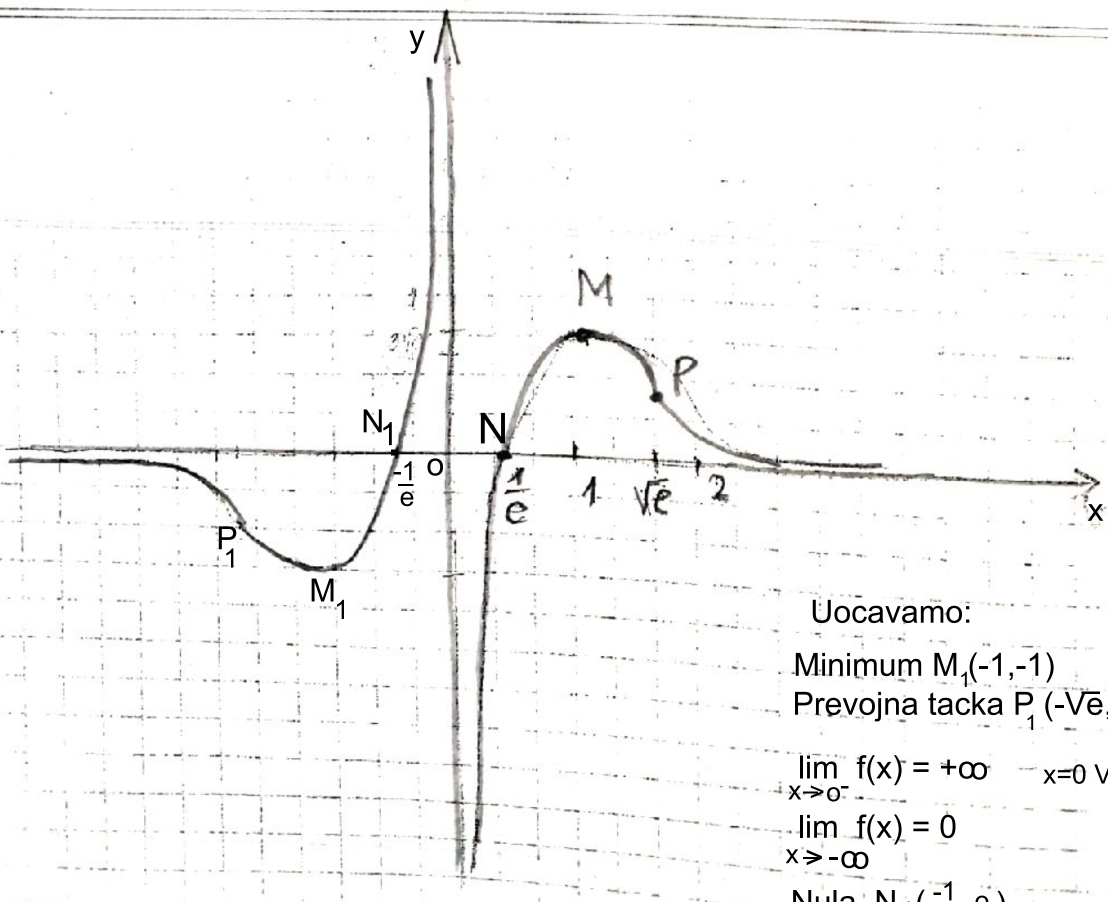
$\iff 2\ln x < \frac{1}{2} \wedge x>0$

$\iff x \in (0, \sqrt{e})$

Prevojna tačka je $P(\sqrt{e}, f(\sqrt{e}))$

$$f(\sqrt{e}) = \frac{\ln\sqrt{e} + 1}{\sqrt{e}} = \frac{\frac{1}{2}+1}{\sqrt{e}} = \frac{3}{2\sqrt{e}} = \frac{3\sqrt{e}}{2e}$$

$P\left(\sqrt{e}, \frac{3\sqrt{e}}{2e}\right)$ $\searrow 0,9$



Uocavamo:

Minimum $M_1(-1,-1)$

Prevojna tacka $P_1\left(-\sqrt{e}, -\frac{3}{2\sqrt{e}}\right)$

$\lim_{x\to 0^-} f(x) = +\infty \quad x=0$ V.A.

$\lim_{x\to -\infty} f(x) = 0$

Nula $N_1\left(\frac{-1}{e}, 0\right)$